Tutorial 5

1. Describe an algorithm to generate a random variable $X|\mu, \sigma^2 \sim N(\mu, \sigma^2)$ where $\mu$ itself is an exponential random variable with parameter $\lambda$ (mean $1/\lambda$). Assume both $\sigma^2$ and $\lambda$ are known parameters.

2. Describe an algorithm to generate a random variable $X|p \sim Binomial(n, p)$ where $p$ itself is a uniform random variable over $[0, 1]$. Assume $n$ is a known positive integer.

3. Use two methods to find out the limit distribution of the Markov Chain with the following transition probability matrix

$$
P = \begin{bmatrix}
0 & \frac{1}{4} & \frac{3}{4} \\
\frac{1}{3} & 0 & \frac{2}{3} \\
\frac{1}{7} & \frac{2}{7} & 0
\end{bmatrix}.
$$

4. Find out the 2-step and 100-step transition probability matrices of $P$ defined in Problem 3, respectively. Compare these two matrices.

5. Write down an algorithm to generate a Markov Chain with the transition probability matrix $P$ defined in Problem 3, using the initial state $X_0 = 1$. 

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