Tutorial 3

1. Give two algorithms for generating a random variable having density function \((\text{Gamma}(2,1))\)
\[ f(x) = x e^{-x}, \quad 0 \leq x < \infty \]
and compare their efficiency.

2. Give algorithms for generating random variables having the probability mass functions
(a)
\[ P(X = i) = \frac{1}{4^i} + \frac{2^i}{3^{i+1}}. \]
(b)
\[ P(X = i) = \frac{1}{2^{i+1}} + \frac{2^{i-2}}{3^i}. \]

3. Give algorithms for generating random variables from the following distributions.
(a)
\[ F(x) = \frac{x + x^3 + x^5}{3}, \quad 0 \leq x \leq 1. \]
(b)
\[ F(x) = \begin{cases} 
\frac{1-e^{-2x}+2x}{3} & \text{if } 0 < x < 1 \\
\frac{3-e^{-2x}}{3} & \text{if } 1 < x < \infty
\end{cases} \]

4. \textbf{(Truncated distributions)} Let \(G\) be a distribution function with density \(g\) and suppose, for constants \(a < b\), we want to generate a random variable from the distribution function
\[ F(x) = \frac{G(x) - G(a)}{G(b) - G(a)} \]
for \(a \leq x \leq b\).

a. If \(X\) has distribution \(G\), then \(F\) is the conditional distribution of \(X\) given what information?

b. Show that the rejection method reduces in this case to generating a random variable \(X\) having distribution \(G\) and then accepting it if it lies between \(a\) and \(b\).

5. Using results of exercise 4, give an algorithm for generating a random variable from the following distribution,
\[ F(x) = \frac{1}{C} \int_a^x \exp\left\{-\frac{(y - \mu)^2}{2\sigma^2}\right\} dy, \quad a \leq x \leq b, \]
where \(C\) is the normalizing constant of \(F(x)\).