Chapter 9

One-way Analysis of Variance
Introduction

- Two independent groups
  - Two-sample t-test
  - Wilcoxon rank-sum test

- Three or more independent groups
  - One-way analysis of variance
  - Kruskal and Wallis test
One-way Analysis of Variance

Example:

- Consider 3 different methods of speed reading
- Assign 15 subjects randomly to the 3 treatment groups, A, B, and C (with 5 subjects per treatment)
- Each of the three groups has received different methods of speed reading instructions
- A reading test is given and the number of words per minute is recorded for each subject.
## Data

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>480</td>
<td>500</td>
</tr>
<tr>
<td>850</td>
<td>460</td>
<td>550</td>
</tr>
<tr>
<td>820</td>
<td>500</td>
<td>480</td>
</tr>
<tr>
<td>640</td>
<td>570</td>
<td>600</td>
</tr>
<tr>
<td>920</td>
<td>580</td>
<td>610</td>
</tr>
</tbody>
</table>
Model

A person’s score =
  the grand mean
+ a component depending on which group the person is in
+ a component depending on the individual’s variability
Model:

\[ y_{ij} = \mu + \alpha_i + \epsilon_{ij} \]

where \( \epsilon_{ij} \sim N(0, \sigma^2) \) independently

Alternatively, we can write the model as follows.
Model: \( y_{ij} = \mu_i + \epsilon_{ij} \)
where \( \mu_i = \mu + \alpha_i \) and \( \epsilon_{ij} \sim N(0, \sigma^2) \) independently
**Sum of Squares**

- Total sum of squares (SST) =

\[ \sum_{i=1}^{3} \sum_{j=1}^{5} (y_{ij} - \bar{y})^2 \]

- Measure the sum of all the square deviations of individual observation from the grand mean.

- It can be shown that SST can be separated into two parts: 
  
  \[ \text{SST} = \text{SSB} + \text{SSE} \]
Sum of Squares

SSB ("Sum of Squares between" for 1-way ANOVA) =

\[
\sum_{i=1}^{3} \sum_{j=1}^{5} (\bar{y}_i - \bar{y})^2 = 5 \sum_{i=1}^{3} (\bar{y}_i - \bar{y})^2,
\]

where \( \bar{y}_i = \sum_{j=1}^{5} y_{ij} / 5 \).

SSE (Sum of Squared Errors) =

\[
\sum_{i=1}^{3} \sum_{j=1}^{5} (y_{ij} - \bar{y}_i)^2
\]
Degrees of Freedom

Associated with each sum of squares is a number called degrees of freedom:

- DF for SST is $N - 1$, where $N$ is the total number of observations.
- DF for SSB is $k - 1 = 2$, where $k$ is the number of groups.
- DF for SSE is $N - k = 12$. 
SSB

- If the treatment means are different from the grand mean, we expect the SSB will be large
- If the treatment means are close to the grand mean, then we expect SSB will be small
- Large SSB may indicate the treatments means are very different
**MSB/MSE**

Two other factors may affect the magnitude of the sum of squares. They are

- the number of observations in each group
- the size of the variation, \( \sigma^2 \)
  - Factor (1) leads us to consider the \( \text{MSB} = \text{SSB} / (k - 1) \)
  - Factor (2) leads us to consider the ratio of MSB divided by the MSE

Note: \( \text{MSE} = \text{SSE} / (N - k) \), which is an estimate of \( \sigma^2 \).
Hypothesis Testing

We want to test $H_0 : \mu_A = \mu_B = \mu_C$

Let $F = \frac{MSB}{MSE} = \frac{SSB/(k-1)}{SSE/(N-k)}$

where $k$ is the number of groups (in our example, it is 3), $N$ is the total number of observations (in our example, it is 15) If $H_0$ is true, then we expect a small observed value of $F$ If $H_0$ is large, then we expect a large observed value of $F$
Hypothesis Testing

• It can be shown that the test statistic, $F$, follows an F-distribution with degrees of freedom $(k - 1)$ and $(N - k)$.

• We reject $H_0$ if the observed $F > F_\alpha(k - 1, N - k)$. 
<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>d.f.</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>SSB</td>
<td>(k-1)</td>
<td>MSB=SSB/ (k-1)</td>
<td>$F_{\text{obs}}$ =MSB/ MSE</td>
<td>Pr($F &gt; F_{\text{obs}}$)</td>
</tr>
<tr>
<td>Within (Error)</td>
<td>SSE</td>
<td>(N-k)</td>
<td>MSE=SSE/ (N-k)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>SST</td>
<td>N-1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Assumptions of ANOVA

- Random samples
- Equal variance
- Independence of errors
- Normal distribution of errors
- Additivity of treatment effects
One-Way ANOVA: SAS

data ex9_1;
input group $ words @@;
datalines;
X 700 X 850 X 820 X 640 X 920
Y 480 Y 460 Y 500 Y 570 Y 580
Z 500 Z 550 Z 480 Z 600 Z 610
;
proc anova data=ex9_1;
title "Analysis of Reading Data";
class group;
model words=group;
means group;
run;
Analysis of Reading Data

The ANOVA Procedure

Dependent Variable: words

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2</td>
<td>215613.3333</td>
<td>107806.6667</td>
<td>16.78</td>
<td>0.0003</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>77080.0000</td>
<td>6423.3333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>14</td>
<td>292693.3333</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R-Square Coeff Var Root MSE words Mean
0.736653 12.98256 80.14570 617.3333

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Anova SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>group</td>
<td>2</td>
<td>215613.3333</td>
<td>107806.6667</td>
<td>16.78</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
## One-Way ANOVA: SAS output

### Analysis of Reading Data

#### The ANOVA Procedure

<table>
<thead>
<tr>
<th>Level of group</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>5</td>
<td>786.000000</td>
<td>113.329003</td>
</tr>
<tr>
<td>Y</td>
<td>5</td>
<td>518.000000</td>
<td>54.037024</td>
</tr>
<tr>
<td>Z</td>
<td>5</td>
<td>548.000000</td>
<td>58.051701</td>
</tr>
</tbody>
</table>
One-Way ANOVA:R

> ex9_1 = read.table("F:/ST2137/leCDATA/ex9_1.txt",header=TRUE)
> attach(ex9_1)
> model1 = aov(words ~ group)
> summary(model1)

            Df Sum Sq Mean Sq F value   Pr(>F)
group        2 215613 107807  16.784 0.0003336 ***
Residuals    12  77080   6423
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
One-Way ANOVA: R

```r
> tapply(words, group, mean)
X Y Z
786 518 548

> mean(words)
[1] 617.3333

> tapply(words, group, mean) - mean(words)
X Y Z
168.6667  -99.3333  -69.3333
```

One-Way ANOVA: R

> boxplot(words~group)
One-Way ANOVA: SPSS

- “Analyze” → “Compare Means” → “One-way ANOVA...”
  
  Note: Variable “Group” may not appear in the left window. You may recode the group variable into a numerical variable.

- Click “Option”

- “Continue” → “OK”
One-Way ANOVA: SPSS output
### One-Way ANOVA: SPSS output

#### Descriptives

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error</th>
<th>95% Confidence Interval for Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Upper Bound</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>786.00</td>
<td>113.930</td>
<td>50.951</td>
<td>644.54</td>
<td>927.46</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>518.00</td>
<td>54.037</td>
<td>24.166</td>
<td>450.90</td>
<td>585.10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>548.00</td>
<td>58.052</td>
<td>25.962</td>
<td>475.92</td>
<td>620.08</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>617.33</td>
<td>144.591</td>
<td>37.333</td>
<td>537.26</td>
<td>697.41</td>
<td></td>
</tr>
</tbody>
</table>

#### ANOVA

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>215613.333</td>
<td>2</td>
<td>107806.667</td>
<td>16.784</td>
<td>.000</td>
</tr>
<tr>
<td>Within Groups</td>
<td>77080.000</td>
<td>12</td>
<td>6423.333</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>292693.333</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Multiple comparison tests

- The F-test in the above example shows that the reading methods are different.
- What are the differences?
- Is X better than Y or Z?
- Are the means of groups Y and Z so close that we cannot consider them different?
- In general, methods used to find group differences after the null hypothesis has been rejected are called post hoc, or multiple-comparison tests.
Some multiple comparison test

- Least significant difference test
- Others
  - Duncan’s multiple-range test
  - Student-Newman-Keul’s multiple-range test
  - Scheffe’s multiple-comparison procedure
Least Significant Difference

- Consider $\mu_i - \mu_j$
- LSD for $\mu_i - \mu_j$ is given by

$$t_{(1-\alpha/2), (N-k)} \sqrt{\text{MSE} \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}$$

where $N$ is the total number of observations, $k$ is the number of groups.

- We conclude $\mu_i$ is different from $\mu_j$ if

$$|\bar{X}_i - \bar{X}_j| > LSD$$
**LSD: SAS**

```sas
proc glm data=ex9_1;
title "Analysis of Reading Data";
class group;
model words=group;
means group/lsd;
run;
```
LSD: SAS output

The GLM Procedure

t Tests (LSD) for words

NOTE: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Error Degrees of Freedom</th>
<th>Error Mean Square</th>
<th>Critical Value of t</th>
<th>Least Significant Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>12</td>
<td>6423.333</td>
<td>2.17881</td>
<td>110.44</td>
</tr>
</tbody>
</table>

Means with the same letter are not significantly different.

<table>
<thead>
<tr>
<th>T Grouping</th>
<th>Mean</th>
<th>N</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>786.00</td>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>B</td>
<td>548.00</td>
<td>5</td>
<td>Z</td>
</tr>
<tr>
<td>B</td>
<td>518.00</td>
<td>5</td>
<td>Y</td>
</tr>
</tbody>
</table>
LSD: R

```r
#compute LSD
> group.1 = words[group == "X"]
> group.2 = words[group == "Y"]
> group.3 = words[group == "Z"]
> group.means = tapply(words, group, mean)
> treat.group = cbind(group.1, group.2, group.3)
> d = 0
> for (i in 1:3)
+ d = d + sum((treat.group[, i] - mean(treat.group[, i]))^2)
> mse = d / 12
> lsd = qt(0.975, 12) * sqrt(mse * 2/5)
```
**LSD: R**

```r
> # Function to check if the difference of 2 means > LSD
> check.lsd = function(obj, i, j, lsd) {
+   mx = mean(obj[, i]); my = mean(obj[, j])
+   d = mx - my
+   if(abs(d) > lsd) cat("There is significant difference between groups", i, "&", j, "\n", "Means = ", mx, ", ", my, "Diff = ", d, " > LSD = ", lsd, " \n" ) else cat("There is no significant difference between groups", i, "&", j, "\n", Means = " , mx, ", " , my, " Diff = " , d, " < LSD = " , lsd, " \n" )
```

30
LSD: R

> check.lsd(treat.group,1,2,lsd)
There is significant difference between groups 1 & 2
Means=786, 518 Diff=268>LSD=110.4410
> check.lsd(treat.group,1,3,lsd)
There is significant difference between groups 1 & 3
Means=786, 548 Diff=238>LSD=110.4410
> check.lsd(treat.group,2,3,lsd)
There is no significant difference between groups 2 & 3
Means=518, 548 Diff=-30<LSD=110.4410
**LSD: SPSS**

- “Analyze” → “Compare Means” → “One-way ANOVA...”
  
  Note: Variable “Group” may not appear in the left window. You may recode the group variable into a numerical variable.

- Click “Post Hoc...” and then choose LSD

- “Continue” → “OK”
LSD: SPSS output
## LSD: SPSS output

### Multiple Comparisons

<table>
<thead>
<tr>
<th>(I) Group Number</th>
<th>(J) Group Number</th>
<th>Mean Difference (I-J)</th>
<th>Std. Error</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>268.000*</td>
<td>50.689</td>
<td>.000</td>
<td>157.56</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>238.000*</td>
<td>50.689</td>
<td>.001</td>
<td>127.56</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-268.000*</td>
<td>50.689</td>
<td>.000</td>
<td>-378.44</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-30.000</td>
<td>50.689</td>
<td>.565</td>
<td>-140.44</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-238.000*</td>
<td>50.689</td>
<td>.001</td>
<td>-348.44</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>30.000</td>
<td>50.689</td>
<td>.565</td>
<td>-80.44</td>
</tr>
</tbody>
</table>
Contrast method

- Contrast of $\mu$’s
  
  For example:  
  
  \[ C_1 = \mu_2 - \mu_3 \]
  
  \[ C_2 = 2\mu_1 - \mu_2 - \mu_3 \]

- Test $H_0$: $C_1 = 0$,
  
  $H_0 : C_2 = 0$

- Contrast: $\sum_{i=1}^{k} a_i \mu_i$ such that $\sum_{i=1}^{k} a_i = 0$. 
Sum of squares contrast

- SSc for contrast \( C = \sum_{i=1}^{k} a_i \mu_i \) is given by

\[
SSc = \left( \sum_{i=1}^{k} a_i \bar{X}_i \right)^2 / \left( \sum_{i=1}^{k} a_i^2 / n_i \right)
\]

- If \( \sum_{i=1}^{k} a_i \mu_i = 0 \) then SSc should be small.

- We can show that under the null hypothesis, \( H_0 : \sum_{i=1}^{k} a_i \mu_i = 0 \), the ratio \( SSc/MSE \) follows an F distribution with degrees of freedom 1 and \( N-k \).

- Reject \( H_0 \) if \( SSc/MSE > F_{0.05}(1,N-k) \)
Contrast method: SAS

```sas
proc glm data=ex9_1;
title “Analysis of Reading Data”;   
class group;
model words=group;
means group;
contrast ‘X vs Y and Z’ group -2 1 1;
contrast ‘Method Y vs Z’ group 0 -1 1;
run;
```
**Contrast method: SAS partial output**

**Analysis of Reading Data**  
19:12 Sunday, August 1, 20

**The GLM Procedure**

**Dependent Variable: words**

<table>
<thead>
<tr>
<th>Contrast</th>
<th>DF</th>
<th>Contrast SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>X vs Y and Z</td>
<td>1</td>
<td>213363.3333</td>
<td>213363.3333</td>
<td>33.22</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Method Y vs Z</td>
<td>1</td>
<td>2250.0000</td>
<td>2250.0000</td>
<td>0.35</td>
<td>0.5649</td>
</tr>
</tbody>
</table>
Contrast method: R

> #multiple comparisons
> #Contrast_1=0*mu_X-1*mu_Y+1*mu_Z
> #Contrast_2=2*mu_X-1*mu_Y-1*mu_Z
> contrasts(group)=matrix(c(0,-1,1,2,-1,-1),nrow=3)
> contrasts(group)

<table>
<thead>
<tr>
<th></th>
<th>[,1]</th>
<th>[,2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Y</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>Z</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

> modelc=aov(words~group)
> summary.lm(modelc)
Contrast method: R

Call:
aov(formula = words ~ group)

Residuals:

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-146</td>
<td>-53</td>
<td>2</td>
<td>57</td>
<td>134</td>
</tr>
</tbody>
</table>

Coefficients:

|            | Estimate | Std. Error | t value | Pr(>|t|) |
|-------------|----------|------------|---------|----------|
| (Intercept) | 617.33   | 20.69      | 29.83   | 1.26e-12 *** |
| group1      | 15.00    | 25.34      | 0.592   | 0.565    |
| group2      | 84.33    | 14.63      | 5.763   | 8.97e-05 *** |

---

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 80.15 on 12 degrees of freedom
Multiple R-squared: 0.7367,  Adjusted R-squared: 0.6928
F-statistic: 16.78 on 2 and 12 DF,  p-value: 0.0003336
Contrast method: SPSS

- “Analyze” → “Compare Means” → “One-way ANOVA...”
- Click “Contrast” and input the coefficients of the contrasts
- “Continue” → “OK”
Contrast method: SPSS output

### Contrast Coefficients

<table>
<thead>
<tr>
<th>Contrast</th>
<th>Group Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2 1 1</td>
</tr>
<tr>
<td>2</td>
<td>0 1 -1</td>
</tr>
</tbody>
</table>

### Contrast Tests

<table>
<thead>
<tr>
<th>Contrast</th>
<th>Value of Contrast</th>
<th>Std. Error</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>words</td>
<td>Assume equal variances</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-506.00</td>
<td>87.795</td>
<td>-5.763</td>
<td>12</td>
<td>.000</td>
</tr>
<tr>
<td>2</td>
<td>-30.00</td>
<td>50.689</td>
<td>-0.592</td>
<td>12</td>
<td>.565</td>
</tr>
<tr>
<td>Does not assume equal variances</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-506.00</td>
<td>107.898</td>
<td>-4.690</td>
<td>4.991</td>
<td>.005</td>
</tr>
<tr>
<td>2</td>
<td>-30.00</td>
<td>35.468</td>
<td>-0.846</td>
<td>7.959</td>
<td>.422</td>
</tr>
</tbody>
</table>
Model Checking: SAS

Residual plots

```sas
proc glm data=ex9_1;
title "Analysis of Reading Data";
class group;
model words=group;
means group;
output out=ex9_1out p=yhat r=resid;
run;
```
Model Checking: SAS

```sas
proc univariate data=ex9_1out;
var resid;
histogram resid/normal;
qqplot resid;
run;
quit;
proc gplot data=ex9_1out;
title "Residual Plots";
plot resid*yhat;
run;
```
Model Checking: SAS partial output

Analysis of Reading Data  19:12 Sun

The UNIVARIATE Procedure
Fitted Normal Distribution for resid

Parameters for Normal Distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Mu</td>
<td>0</td>
</tr>
<tr>
<td>Std Dev</td>
<td>Sigma</td>
<td>74.2005</td>
</tr>
</tbody>
</table>

Goodness-of-Fit Tests for Normal Distribution

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolmogorov-Smirnov</td>
<td>D 0.15828730</td>
<td>&gt;0.</td>
</tr>
<tr>
<td>Cramer-von Mises</td>
<td>W-Sq 0.05535680</td>
<td>&gt;0.</td>
</tr>
<tr>
<td>Anderson-Darling</td>
<td>A-Sq 0.32488548</td>
<td>&gt;0.</td>
</tr>
</tbody>
</table>

Quantiles for Normal Distribution

<table>
<thead>
<tr>
<th>Percent</th>
<th>Observed</th>
<th>Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-146.00000</td>
<td>-172.616177</td>
</tr>
</tbody>
</table>
Model Checking: SAS partial output
Model Checking: R

```r
mcheck=function(obj){
  rs=obj$resid
  fv=obj$fitted
  par(mfrow=c(2,1))
  plot(fv,rs,xlab="Fitted values",ylab="Residuals")
  abline(h=0,lty=2)
  qqnorm(rs,xlab="normal scores",ylab="ordered residuals")
  qqline(rs,lty=2)
  par(mfrow=c(1,1))
}
mcheck(modelc)
```
Nonparametric 1-way Test: SAS

Kruskal-Wallis Test

```sas
proc npar1way data=ex9_1;
class group;
var words;
exact wilcoxon;
run;
```
Kruskal-Wallis Test: SAS output

Residual Plots

The NPAR1WAY Procedure

Wilcoxon Scores (Rank Sums) for Variable words
 Classified by Variable group

<table>
<thead>
<tr>
<th>group</th>
<th>N</th>
<th>Sum of Scores</th>
<th>Expected Under H0</th>
<th>Std Dev Under H0</th>
<th>Mean Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>5</td>
<td>65.0</td>
<td>40.0</td>
<td>8.150372</td>
<td>13.00</td>
</tr>
<tr>
<td>Y</td>
<td>5</td>
<td>23.0</td>
<td>40.0</td>
<td>8.150372</td>
<td>4.60</td>
</tr>
<tr>
<td>Z</td>
<td>5</td>
<td>32.0</td>
<td>40.0</td>
<td>8.150372</td>
<td>6.40</td>
</tr>
</tbody>
</table>

Average scores were used for ties.

Kruskal-Wallis Test

Chi-Square         9.8151
DF                  2
Asymptotic Pr > Chi-Square  0.0074
Exact Pr >= Chi-Square  0.0012
Kruskal-Wallis Test: R

> kruskal.test(words, group)
  
  Kruskal-Wallis rank sum test

data:  words and group

Kruskal-Wallis chi-squared = 9.8151, df = 2, p-value = 0.007391
Kruskal-Wallis Test: SPSS

- “Analyze” → “Nonparametric Tests” → “Legacy Dialogs” → “K independent Samples”

- Move variable “words” to “Test variable list” window and “groupno” to the “Grouping Variable” window

- Click “Define Range” and input the minimum the maximum group number

- “Continue” → “OK”
Kruskal-Wallis Test: SPSS output
Kruskal-Wallis Test: SPSS output

<table>
<thead>
<tr>
<th>Ranks</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group Number</td>
<td>N</td>
<td>Mean Rank</td>
</tr>
<tr>
<td>words</td>
<td>1</td>
<td>5</td>
<td>13.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>4.60</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>6.40</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

Test Statistics\(^{a,b}\)

<table>
<thead>
<tr>
<th></th>
<th>words</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-square</td>
<td>9.815</td>
</tr>
<tr>
<td>df</td>
<td>2</td>
</tr>
<tr>
<td>Asymp. Sig.</td>
<td>.007</td>
</tr>
</tbody>
</table>

\(^a\) Kruskal Wallis Test

\(^b\) Grouping Variable: Group Number