Summary of Chapter 3

1. Conditional probability
   \[ P(E \mid F) = \frac{P(EF)}{P(F)} \]

2. Multiplication rule
   \[ P(E_1, E_2, \ldots, E_n) = P(E_1)P(E_2 \mid E_1) \ldots P(E_n \mid E_1, \ldots, E_{n-1}) \]

3. Theorem of total probability
   \[ P(E) = P(E \mid F)P(F) + P(E \mid F^c)P(F^c) \]
   \[ P(E) = \sum_{i=1}^{n} P(E \mid F_i)P(F_i) \]

4. Bayes' formula
   \[ P(F_j \mid E) = \frac{P(E \mid F_j)P(F_j)}{\sum_{i=1}^{n} P(E \mid F_i)P(F_i)} \]
Summary of Chapter 3

5. E and F independent
   \[ P(E|F) = P(E) \]
   \[ P(EF) = P(E) \cdot P(F) \]

6. \( E_1, E_2, \ldots, E_n \) are independent if for any subset \( E_{i_1}, E_{i_2}, \ldots, E_{i_r} \) of them
   \[ P(E_{i_1}E_{i_2}\ldots E_{i_r}) = P(E_{i_1}) \cdot P(E_{i_2}) \cdots P(E_{i_r}) \]

7. For a fixed event F, \( P(E|F) \) can be considered to be a probability function on the events E of the sample space.
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