Solution to TUTORIAL 10

**Solution to Q10.1:** To compare between large units and small units in simple random sampling, we need to compare the following variances (on small unit basis):

\[
S_S^2 = \frac{1}{2499} \sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{..})^2
\]

\[
S_L^2 = \frac{1}{499 \times 5} \sum_{i=1}^{10} \sum_{j=1}^{50} (5\bar{Y}_{ij}. - 5\bar{Y}_{..})^2
\]

\[
= \frac{5}{499} \sum_{i=1}^{10} \sum_{j=1}^{50} (\bar{Y}_{ij}. - \bar{Y}_{..})^2.
\]

To compare between large units and small units in stratified random sampling, we need to compare the following variances (on small unit basis):

\[
S_{stS}^2 = \frac{1}{10 \times 249} \sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{i.})^2
\]

\[
S_{stL}^2 = \frac{1}{10 \times 49 \times 5} \sum_{i=1}^{10} \sum_{j=1}^{50} (5\bar{Y}_{ij}. - 5\bar{Y}_{i.})^2
\]

\[
= \frac{5}{10 \times 49} \sum_{i=1}^{10} \sum_{j=1}^{50} (\bar{Y}_{ij}. - \bar{Y}_{i.})^2.
\]

In the above, the averages are defined as follows:

\[
\bar{Y}_{..} = \frac{1}{10 \times 50 \times 5} \sum_{i=1}^{1} \sum_{j=1}^{50} \sum_{k=1}^{5} Y_{ijk},
\]

\[
\bar{Y}_{i.} = \frac{1}{50 \times 5} \sum_{j=1}^{50} \sum_{k=1}^{5} Y_{ijk},
\]

\[
\bar{Y}_{ij} = \frac{1}{5} \sum_{k=1}^{5} Y_{ijk}.
\]

What are given are the following quantities:

1
\[ \text{MSB}_{st} = \frac{50 \times 5}{9} \sum_{i=1}^{10} (\bar{Y}_{i..} - \bar{Y}_{..})^2 = 30.6 \]
\[ \text{MSW}_{stL} = \frac{5}{490} \sum_{i=1}^{10} \sum_{j=1}^{50} (\bar{Y}_{ij} - \bar{Y}_{..})^2 = 3.0 \]
\[ \text{MSW}_{i..} = \frac{1}{2000} \sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{ij..})^2 = 1.6. \]

From the above given quantities, we can compute
\[
\sum_{i=1}^{10} \sum_{j=1}^{50} (Y_{ij} - \bar{Y}_{..})^2 \\
= \sum_{i=1}^{10} \sum_{j=1}^{50} (Y_{ij} - \bar{Y}_{i..})^2 + 50 \sum_{i=1}^{10} (Y_{i..} - \bar{Y}_{..})^2 \\
= 349.08 \\
\sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{i..})^2 \\
= \sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{ij..})^2 + 5 \sum_{i=1}^{10} \sum_{j=1}^{50} (Y_{ij} - \bar{Y}_{i..})^2 \\
= 4670 \\
\sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{..})^2 \\
= \sum_{i=1}^{10} \sum_{j=1}^{50} \sum_{k=1}^{5} (Y_{ijk} - \bar{Y}_{ij..})^2 + (50 \times 5) \sum_{i=1}^{10} (Y_{i..} - \bar{Y}_{..})^2 \\
= 4945.4.
\]

Finally, we obtain
\[ S_g^2 = 1.9789 \quad S_{L}^2 = 3.497796 \]
\[ S_{stS}^2 = 1.8755 \quad S_{stL}^2 = 3. \]

Hence, in the simple random sampling case, the relative efficiency for large units is \(1.9789/3.497796 = 0.5657563\), and in the stratified case, the relative efficiency is \(1.8755/3 =0.6251667\).
Solution to Q10.2: Hint: Express $S_3^2$, $S_L^2$, $S_{stS}^2$ and $S_{stL}^2$ in terms of $S_1^2$, $S_2^2$ and $S_3^2$, then compare the ratios $S_{stS}^2/S_{stL}^2$ and $S_3^2/S_L^2$. 