1. Let $F_n, n = 0, 1, 2, \ldots$, be cumulative distribution functions such that $F_n \to F_0$ for every continuity point of $F_0$. Let $U$ be a random variable having the uniform distribution on the interval $[0, 1]$ and let $G_n(U) = \sup\{x : F_n(x) \leq U\}, n = 0, 1, 2, \ldots$. Show that $G_n(U) \to_p G_0(U)$.

2. Let $X_n$ be a random variable distributed as $\mathcal{N}(\mu_n, \sigma_n^2), n = 1, 2, \ldots$, and $X$ be a random variable distributed as $\mathcal{N}(\mu, \sigma^2)$. Show that $X_n \to_d X$ if and only if $\lim_n \mu_n = \mu$ and $\lim_n \sigma_n^2 = \sigma^2$.

3. Let $U_1, U_2, \ldots$ be independent random variables having the uniform distribution on $[0, 1]$ and $Y_n = (\prod_{i=1}^n U_i)^{-1/n}$. Show that $\sqrt{n}(Y_n - e) \to_d N(0, e^2)$.

4. Suppose that $X_n$ is a random variable having the binomial distribution with size $n$ and probability $\theta \in (0, 1), n = 1, 2, \ldots$. Define $Y_n = \log(X_n/n)$ when $X_n \geq 1$ and $Y_n = 1$ when $X_n = 0$. Show that $\lim_n Y_n = \log \theta$ a.s. and $\sqrt{n}(Y_n - \log \theta) \to_d N(0, \frac{1-\theta}{\theta\sigma^2})$.

5. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with $\text{Var}(X_1) < \infty$. Show that

$$\frac{2}{n(n+1)} \sum_{j=1}^n jX_j \to_p EX_1.$$ 

6. Let $X, X_1, X_2, \ldots$ be random $k$-vectors and $Y, Y_1, Y_2, \ldots$ be random $l$-vectors. Suppose that $X_n \to_d X$, $Y_n \to_d Y$, and $X_n$ and $Y_n$ are independent for each $n$. Show that $(X_n, Y_n)$ converges in distribution to a random $(k+l)$-vector.

7. Let $X, X_1, X_2, \ldots$ be random $k$-vectors and $A_1, A_2, \ldots$ be events. Suppose that $X_n \to_d X$. Show that $X_n I_{A_n} \to_d X$ if and only if $P(A_n) \to 1$. 
