1. In an experiment to evaluate the effect of diets on mammary carcinogenesis risk of rats. A sample of rats were treated with the drug DMBA. Starting 6 weeks after DMBA administration, each rat was examined once weekly for 14 weeks. The survival time of concern is the time until a tumor is detected after the DMBA administration. Describe in detail the types of censoring that are represented by the following rats, and provide the survival time or censoring time data in days:

(a) A rat that had a tumor at the first examination at 6 weeks.

(b) A rat that survived the study without having any tumors.

(c) A rat that did not have tumor at week 12 but had tumor at week 13.

(d) A rat that died (without tumor present and death was unrelated to the occurrence of cancer) at day 37.

2. A large number of disease-free individuals were enrolled in a study beginning January 1, 1970, and were followed for 30 years to assess the age at which they developed breast cancer. Individuals had clinical exams every 3 years after enrollment. For selected individuals described below, discuss the types of censoring and truncation that are represented and provide their data (survival time or censoring time).

(a) A healthy woman, enrolled in the study at age 30, never developed breast cancer during the study.

(b) A healthy woman, enrolled in the study at age 40, was diagnosed with breast cancer at the fifth exam after enrollment.

(c) A healthy woman, enrolled in the study at age 50, died from a cause unrelated to the disease at age 61.

(d) A healthy woman, enrolled in the study at age 42, moved away from the community at age 55 and was never diagnosed with breast cancer during the period of observation.
3. To estimate the distribution of the ages at which postmenopausal women develop breast cancer, a sample of eight 50-year old women were yearly mammograms for a period of 10 years. At each exam, the presence or absence of breast cancer was recorded. In the study, no tumors were detected by the women by self-examination between two scheduled yearly exams. Four of the eight women were not detected with breast cancer during the study period. The ages at onset of breast cancer for the eight women were in the following intervals:

\( (55, 56], (58, 59], (52, 53], (59, 60], \geq 60, \geq 60, \geq 60, \geq 60, \geq 60, \geq 60 \).

What type of censoring or truncation is represented in this example?

4. In the survey concerning the age of first use of marijuana, data was collected by asking the question: “when did you first use marijuana?” The answers of four persons are in the following:

(a) Person A, aged 18: “I first used marijuana at age 16.”
(b) Person B, aged 17: “I used marijuana but I forget at what age.”
(c) Person C, aged 17: “I never use marijuana.”
(d) Person D, aged 19: “I first used marijuana in secondary school but I forget which year.”

For each of the four persons, discuss whether there is a censoring or truncation, if there is, what type? Provide the survey data.

5. Suppose that the time to death \( X \) has an exponential distribution with hazard rate \( \lambda \) and that the right-censoring time \( C \) is exponential with hazard rate \( \theta \). Let \( T = \min(X, C) \) and \( \delta = 1 \) if \( X \leq C \); 0, if \( X > C \). Assume that \( X \) and \( C \) are independent.

(a) Find \( P(\delta = 1) \).

(b) Find the distribution of \( T \).

(c) Show that \( \delta \) and \( T \) are independent.

(d) Let \((T_1, \delta_1), \ldots, (T_n, \delta_n)\) be a random sample from this model. Show that the maximum likelihood estimator of \( \lambda \) is \( \sum_{i=1}^n \delta_i / \sum_{i=1}^n T_i \). Use part (a)-(c) to find the mean and variance of \( \hat{\lambda} \).