1. Assume that \( Y_i = Z_i / m_i, i = 1, \ldots, n \), where \( Z_i \)'s are independent with distribution \( Bi(m_i, \pi_i) \). Consider the GLIM with the canonical link

\[
\eta = \log\left( \frac{\pi}{1 - \pi} \right)
\]

and linear predictor

\[
\eta_i = x_i' \beta.
\]

Derive the expression for the diagonal elements \( w_i \) of the weight matrix and the component \( z_i \) of the pseudo-response vector \( z \) in the IWLS procedure for the estimation of \( \beta \).

2. Let \( y_i, \hat{\mu}_i, i = 1, \ldots, n \), be the observations and estimated mean values from the fitting of the GLIM with the following distributions:

   (i) Normal \((\mu, \sigma^2)\);
   (ii) Gamma \((\mu, \nu)\);
   (iii) Inverse Gaussian \((\mu, \sigma^2)\).

Derive the deviance for each of the above distributions.

3. Derive the expressions of the Pearson residual, the working residual and the deviance residual for the GLIM with Poisson, Binomial and Gamma distributions, respectively.

4. Suppose that \( Y_1, \cdots, Y_m \) are independent Bernoulli random variables for which

\[
P(Y_i = 0) = 1 - \pi \quad \text{and} \quad P(Y_i = 1) = \pi.
\]

Show that any fixed sequence comprising \( y \) ones and \( m - y \) zeros has probability \( \pi^y (1 - \pi)^{m - y} \). Hence deduce that the total \( Y = \sum_{i=1}^{m} Y_i \) has the binomial distribution with index \( m \) and probability of success \( \pi \).
5. Suppose that $Y_1 \sim B(m_1, \pi)$ and $Y_2 \sim B(m_2, \pi)$ are independent. Deduce that $Y = Y_1 + Y_2 \sim B(m, \pi)$, where $m = m_1 + m_2$. 