1. The following three questions pertain to different aspects of the multiple comparison artifact.

(a) If each of $K$ contrasts was tested for significance at the $\alpha$ significance level, how many contrasts would you expect, just by chance, to find significant? What is this expected number when $K = 100$ and $\alpha = 0.05$? vsm

About $K\alpha$ contrasts would be expected to be found significant. For $K = 100$, about 5 will be bound significant.

(b) Suppose that the ratios $\hat{C}/\text{se}(\hat{C})$ for each of $K$ contrasts are mutually independent and that each is tested for significance at the $\alpha$ significance level. Show that the probability that at least one is erroneously found to be significant is equal to $1 - (1 - \alpha)^K$. What is this probabilty if $K = 14$ and $\alpha = 0.05$?

Let $A_k$ denote the event that the $k$th contrast is erroneously found to be significant. The probability that at least one is erroneously found to be significant is given by

$$P(\bigcup_{k=1}^{K} A_k) = 1 - P(\bigcap_{k}^{K} A_k^C) = 1 - \prod_{k}^{K} P(A_k^C) = 1 - (1 - \alpha)^K.$$  

For $K = 14$ and $\alpha = 0.05$, this probability is given by

$$1 - 0.95^{14} = 0.512325.$$  

(c) Investigators will occasionally single out the smallest and the largest of $g$ means for a $t$ test at the $\alpha$ significance level. That is, when the sample sizes are all equal to a common $n$, the largest and smallest means are declared to differ significantly if

$$L = \frac{\max \bar{X}_i - \min \bar{X}_i}{\sqrt{\text{WMS}}} \sqrt{\frac{n}{2}}$$

exceeds $t_{n-g,\alpha/2}$. However, the correct critical values of $L$ are given by those of the studentized range distribution divided by $\sqrt{2}$. Suppose that $g = 4, n = 7$ and $L = 2.1$. 

1
What would the verdict be, significant or not, if $L$ was compared to $t_{24,0.025}$? Check that $L$ would have to exceed 2.76 in order for significance to be declared at the same level using the correct critical value.

$t_{24,0.025} = 2.063899$. Since $L = 2.1 > 2.063899$, the contrast will be declared significant. Since the $L$ above is the studentized range divided by $\sqrt{2}$ and the critical value of the studentized range is $q_{4,24} = 3.901$, for $L$ to be declared significant according to correct critical values, $L$ must be larger than $q_{4,24}/\sqrt{2} = 2.7584$.

2. The following table summarizes the data from a clinic trial for the comparison of two drugs to a control. The response variable is a blood count (in millions of cells per cubic millimeter).

<table>
<thead>
<tr>
<th>Group</th>
<th>$n_i$</th>
<th>$\bar{y}_i$</th>
<th>$s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>6</td>
<td>8.25</td>
<td>0.94</td>
</tr>
<tr>
<td>Drug A</td>
<td>4</td>
<td>8.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Drug B</td>
<td>5</td>
<td>10.88</td>
<td>1.56</td>
</tr>
</tbody>
</table>

where $s_i^2 = \frac{1}{n_i-1} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2$.

(i) Using the summary data, compute the entries of the ANOVA table:

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drug</td>
<td>2</td>
<td>19.737</td>
<td>9.8685</td>
<td>7.1414</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>16.5824</td>
<td>1.3819</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
<td>36.3194</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Computation for $SSTr$ (i.e., sum of squares due to drugs):

\[
\bar{y}_\cdot = \frac{1}{N} \sum_i n_i \bar{y}_i = \frac{1}{15} (6 \times 8.25 + 4 \times 8.9 + 5 \times 10.88) = 9.3.
\]

\[
SSTr = \sum_i n_i (\bar{y}_i - \bar{y}_\cdot)^2 = 19.737.
\]

(b) Computation for $SSE$:

\[
SSE = \sum_i \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2 = \sum_i (n_i - 1) s_i^2
\]

\[
= 5 \times 0.94^2 + 3 \times 0.9^2 + 4 \times 1.56^2 = 16.5824.
\]

(c) The computation of other items are straightforward.
(ii) Conduct an appropriate test for the null hypothesis that there is no difference in the effects of the three drugs at level $\alpha = 0.05$.

The statistic for the F-test is $F = 7.1414$. Under the null hypothesis, the F-statistic has a $F$-distribution with d.f. 2 and 12. The p-value is $P(F_{2,12} \geq 7.1414) = 0.00905853$. Since the p-value is smaller than 0.05, the null hypothesis is rejected at this level.

(iii) Using an appropriate multiple comparison method, test whether or not each of Drug A and Drug B is significantly different from the control drug at level $\alpha = 0.05$.

The Dunnett’s multiple comparison is appropriate. The test statistics are:

\[
T_A = \frac{\bar{y}_A - \bar{y}_C}{\sqrt{\text{MSE}(\frac{1}{n_C} + \frac{1}{n_A})}} = \frac{8.9 - 8.25}{\sqrt{1.3819(\frac{1}{6} + \frac{1}{4})}} = 0.8566155.
\]

\[
T_B = \frac{\bar{y}_B - \bar{y}_C}{\sqrt{\text{MSE}(\frac{1}{n_C} + \frac{1}{n_B})}} = \frac{10.88 - 8.25}{\sqrt{1.3819(\frac{1}{6} + \frac{1}{5})}} = 3.6947662.
\]

The two-sided critical value at level 0.05 is $d_{0.05}(2,12) = 2.50$.

Drug A is significantly different from the control drug, but Drug B is not.

R-function for the computation of p-value:

\[
p.value = 1 - \text{pf}(7.1414, 2, 12)
\]

3. For the meat storage example given in Lecture notes 3 & 4, make a pairwise comparison among the four storing conditions.

(i) Find the pairs whose differences are significant at level $\alpha = 0.05$.

From Lecture notes 3 & 4, we have $n_i = 3, i = 1, \ldots, 4$ and

<table>
<thead>
<tr>
<th></th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>3</td>
<td>32.873</td>
<td>10.958</td>
<td>94.584</td>
</tr>
<tr>
<td>Error</td>
<td>8</td>
<td>0.927</td>
<td>0.116</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>33.800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\bar{y}_1 = 7.48, \bar{y}_2 = 5.50, \bar{y}_3 = 7.26, \bar{y}_4 = 3.36
\]
Using the formula

\[ T_{ij} = \sqrt{\frac{n}{2} \frac{\bar{y}_i - \bar{y}_j}{\text{MSE}}} \]

the pairwise t-statistics are computed as:

\[ T_{12} = 0.712003, T_{13} = 0.7911, T_{14} = 14.8154, \]
\[ T_{23} = -6.3289, T_{24} = 7.6954, T_{34} = 14.0243. \]

The critical value is given by \( q_{0.05}(4, 8)/\sqrt{2} = 4.26/\sqrt{2} = 3.2031. \) \( T_{14}, T_{23}, T_{24} \) and \( T_{34} \) are significant at level 0.05.

(ii) Construct simultaneous 95\% confidence intervals for the differences \( \mu_i - \mu_j, \) \( 1 \leq i < j \leq 4, \)
where \( \mu_i \) is the expected mean response under condition \( i. \)

The simultaneous confidence intervals are computed using the formula:

\[ (\bar{y}_i - \bar{y}_j) \pm q_{0.05}(4, 8) \sqrt{\frac{\text{MSE}}{n}}. \]

The intervals are given as follows:

- [12]: [1.0892, 2.8707]
- [13]: [-0.67077, 1.1108]
- [14]: [3.2292, 5.0108]
- [23]: [-2.6507, -0.8692]
- [24]: [1.2492, 3.0307]
- [34]: [3.0092, 4.7908]

4. Consider the following summary data for the yield of tomatoes (kg/plot) for four different levels of salinity; salinity level here refers to electrical conductivity (EC), where the chosen levels were EC = 1.6, 3.8, 6.0, and 10.2 nmhos/cm:

<table>
<thead>
<tr>
<th>EC levels</th>
<th>1.6</th>
<th>3.8</th>
<th>6.0</th>
<th>10.2</th>
</tr>
</thead>
<tbody>
<tr>
<td># of observations</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>mean:</td>
<td>59.53</td>
<td>55.4</td>
<td>50.85</td>
<td>45.5</td>
</tr>
<tr>
<td>sd:</td>
<td>3.232</td>
<td>2.665</td>
<td>2.426</td>
<td>2.901</td>
</tr>
</tbody>
</table>

Assume the true mean yield tomatoes for the specified levels of salinity are \( \mu_1, \mu_2, \mu_3 \) and \( \mu_4 \) respectively.
(i) Calculate the ANOVA $F$-statistic and conclude whether it is significant at level 0.05, using the information provided in the question.

*By the similar calculation to Question 2, it is computed that*

\[
\bar{y}_r = 52.82, \text{SST}_r = 436.5752, \text{SSE} = 95.54798 \\
F = 18.27669, p\text{-value} = 9.040307e^{-05}.
\]

*It is significant.*

(ii) Using an appropriate multiple comparison method, conclude whether or not any of the pairwise differences $\mu_i - \mu_j, i, j = 1, \ldots, 4, i \neq j$ is statistically significant at level 0.05. If yes, what are those pairwise differences?

*Using the formula*

\[
T_{ij} = \frac{\bar{y}_i - \bar{y}_j}{\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}},
\]

*the pairwise t-statistics are computed as:

\[
T_{12} = 2.069879, T_{13} = 4.350254, T_{14} = 7.031574, \\
T_{23} = 2.280375, T_{24} = 4.961695, T_{34} = 2.681320.
\]

*The critical value is given by $q_{0.05}(4, 12)/\sqrt{2} = 4.20/\sqrt{2} = 2.969848.$ $T_{13}, T_{14}$ and $T_{24}$ are significant.*

(iii) Construct a 95% confidence interval for the contrast: $\mu_1 - \frac{\mu_2 + \mu_3 + \mu_4}{3}$. The confidence interval is given by

\[
\hat{\mu}_1 - \frac{\hat{\mu}_2 + \hat{\mu}_3 + \hat{\mu}_4}{3} \pm t_{12,0.025} \sqrt{\frac{MSE}{n} \left(1 + 3\left(\frac{1}{3}\right)^2\right)} \\
= [5.397067, 12.496267]
\]