1. Let $F$ be a c.d.f. on $\mathcal{R}$. Show that
   
   (i) $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$;
   
   (ii) $F(\infty) = \lim_{x \to \infty} F(x) = 1$;
   
   (iii) $F$ is nondecreasing, i.e., $F(x) \leq F(y)$ if $x \leq y$;
   
   (iv) $F$ is right continuous, i.e., $\lim_{y \to x, y>x} F(y) = F(x)$.

2. Let $(\Omega, \mathcal{F})$ be a measurable space. A set function $\nu$ on $\mathcal{F}$ is called a signed measure if (a) $\nu(\emptyset) = 0$ where $\emptyset$ is the empty set and (b) for any disjoint subsets $A_i \in \mathcal{F}, i = 1, 2, \ldots, \nu(\bigcup_i A_i) = \sum_i \nu(A_i)$.
   
   (i) Let $\mu_1$ and $\mu_2$ be two measures on $(\Omega, \mathcal{F})$. If $\mu_1(A) - \mu_2(A)$ is well defined for all $A \in \mathcal{F}$, show that $\mu_1 - \mu_2$ is a signed measure.
   
   (ii) Suppose that $f$ is a measurable function on $(\Omega, \mathcal{F}, \mu_1)$ whose integral exists. Show that $\nu(A) = \int_A f d\mu_1$, $A \in \mathcal{F}$, is a signed measure on $(\Omega, \mathcal{F})$. 