Problem with the Neyman-Pearson Formulation

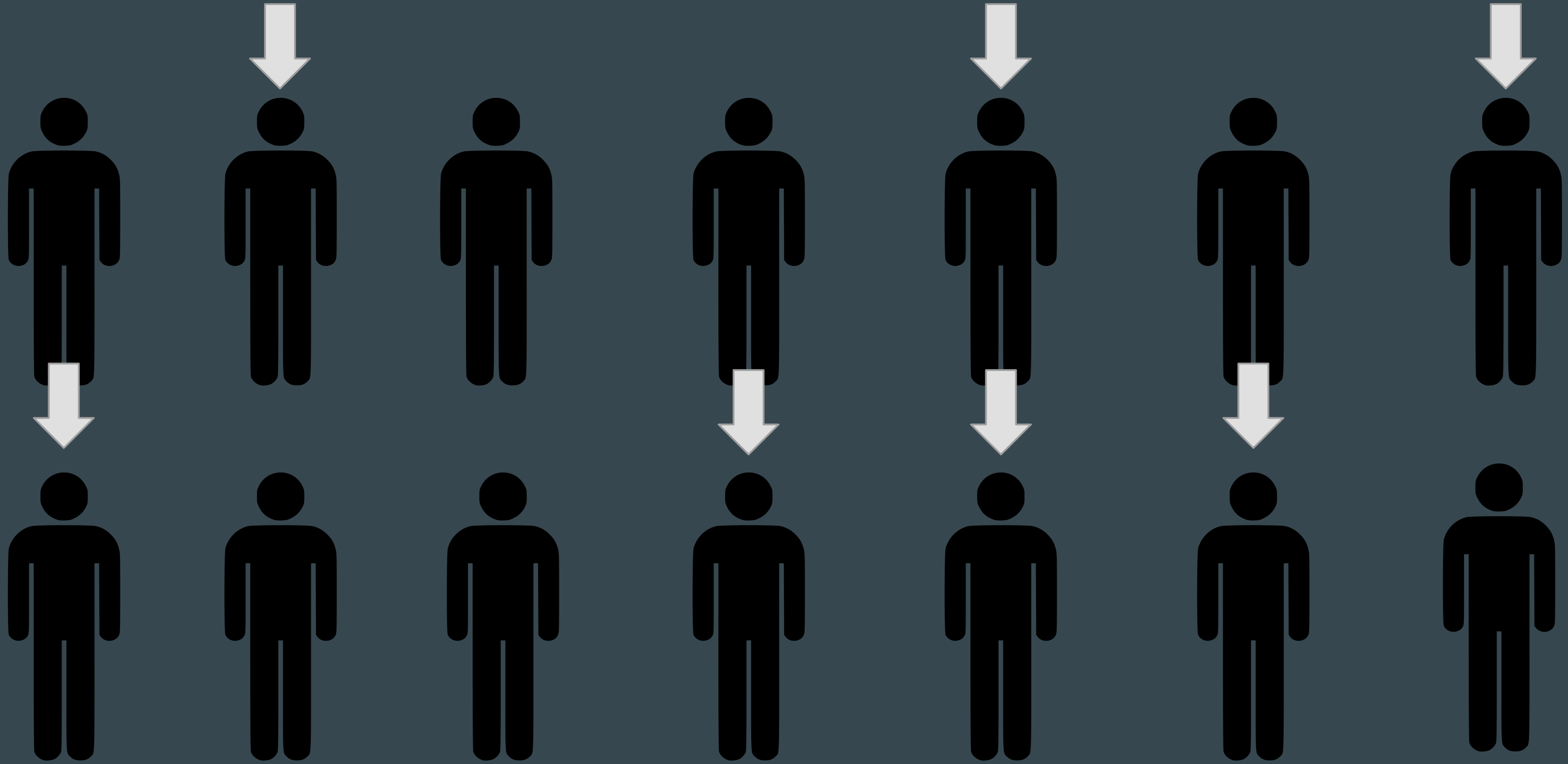
Anabella Fong, Goh Jie Sheng, Bok Wen Xuan

Richard Peto

Fishers Experimental design

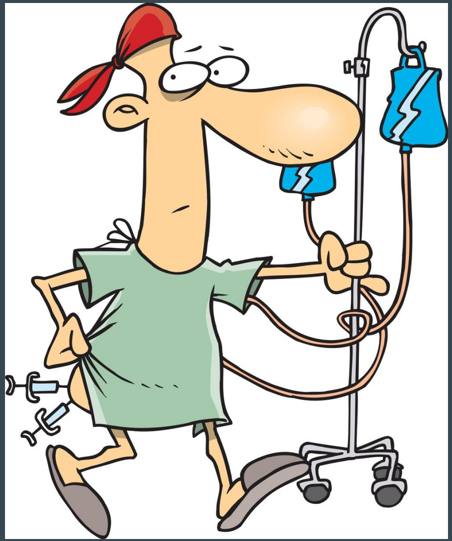
- Randomized selection of patients for treatment
- Results were compared between treatment and non-treatment groups

Randomized selection



Problem with Fisher's experimental design

Randomized
Treatment



Patients

Problem with Fisher's experimental design

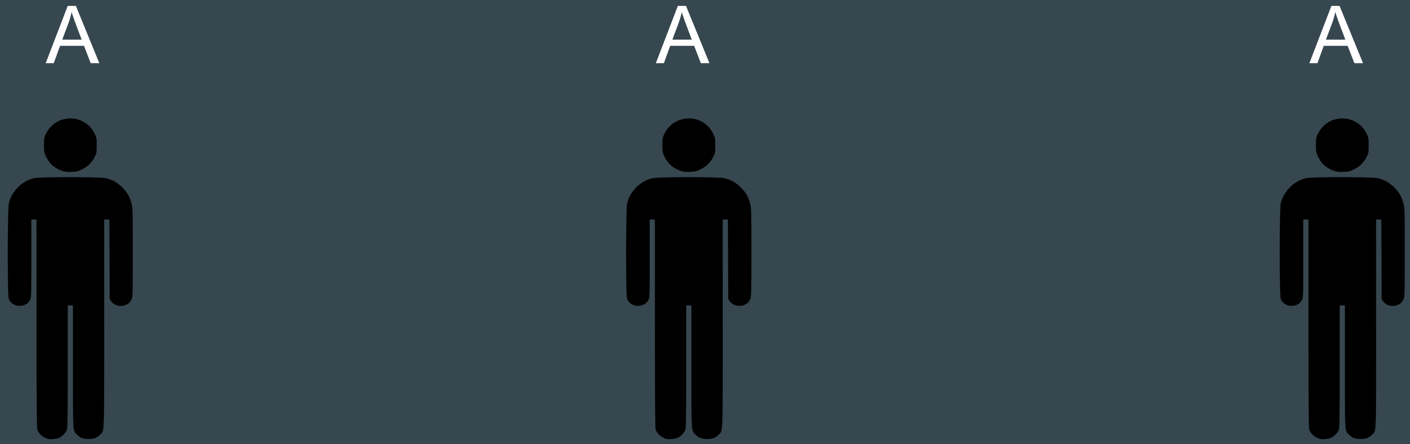
HUMANS!!!!

EXPERIMENTS

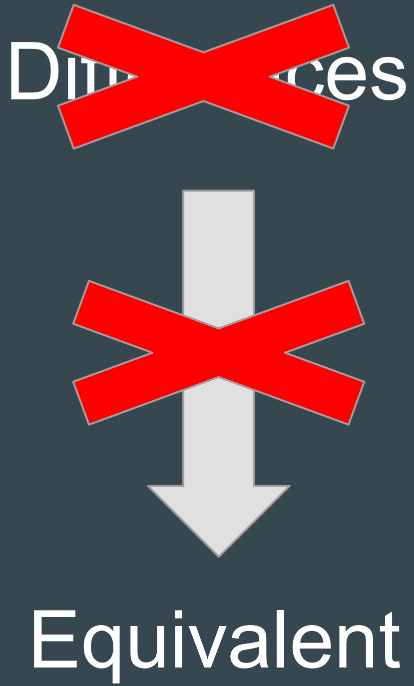
Peto’s ‘intent to treat’

Peto’s ‘intent to treat’

- Patients were randomly selected to receive specific treatment
- Each patient is treated as if they are on their assigned treatment; all changes in treatment are ignored



Problem with Peto’s solution



Problem: Rigidity of Neyman-Pearson Formulation

Neyman-Pearson Formulation

$$\Lambda(x) = \frac{L(x \mid \theta_0)}{L(x \mid \theta_1)} \leq \eta$$

where

$$P(\Lambda(X) \leq \eta \mid H_0) = \alpha$$

Hypothesis testing is the probability that the test correctly rejects null hypothesis as much as possible, when the alternate is true.

Neyman-Pearson Formulation

P-value $\leqslant \alpha$ → Reject Null Hypothesis

P-value $> \alpha$ → Do not reject Null Hypothesis

Fishers problem with the formulation

- No rigorous requirements should be subjected to the use of p-values and significant testing.
- Disagree with fixing α and act only if p-value is less than that.

Cox’s Formulation

- Fisher’s method - significance testing
- Neyman-Pearson Formulation - hypothesis testing
- Hypothesis testing is used for refining the scientist’s views of reality by eliminating unnecessary parameters or for deciding between two differing models of reality.

Box's Approach

- Agreed with Cox's formulation
- Data from one experiment is compared to other experiments
- Using the newer studies, scientist always return to older studies to refine their interpretation

Box's Approach: Example

Manufacturer

+ EVOP

Modifying humidity, speed, sulfur and temperature → Little change in paper strength.

Box’s Approach: Example

Slight differences + Fisher’s analysis of variance → New experiment!

↓

Average strength across all runs has been increased slightly → New run with another slight increase in strength → Slight differences

Cochran’s observational studies

- The city of Baltimore - if public housing had an effect on the social attitudes and progress of poor people
- Following Fisher's methods - take a group of people, whether they had applied for public housing or not, and randomly assign some of them to public housing and refuse it to the others.
- **Observational study -** following families who went into public housing and those who did not
- Once the parameters of all these effects had been estimated, the remaining differences in effect would be used to determine the effect of public housing on social attitudes

Cochran’s observational studies

Cochran's methods : to estimate the underlying effect of treatment, taking into account the effect of imbalances in the assignment of treatment to patients.

Rubin’s Model

- Each patient is assumed to have a possible response to each of the treatments.
- We can set up a mathematical model in which there is a symbol in the formula for each of those possible responses. Rubin derived conditions on this mathematical model that are needed to estimate what might have happened had the patient been put on the other treatment.

Problem with Cochran and Rubin's Model

- Cochran and Rubin methods are **highly model specific** - they will not produce correct answers unless the complicated mathematical models they use come close to describing reality; require the analyst to devise a mathematical model that will match reality in all or most of its aspects.

Conclusion

- No mathematical model is perfect
- Need to know what we are looking for to design a good model for it