Summary

The objectives of this report are to find good estimators of parameters in regression model with auto-regressive error and investigate properties of those estimators. Meanwhile, this report will discuss the relative advantages of regression model with auto-regressive error by comparing it with common regression models. Experiment results show that regression model with auto-regressive error produces more accurate estimation, especially when explanatory variables x_i has $I(0)$ and $I(1)$ preocess.

We usually describe relationship between two variables by a linear regression model: $y_i = \alpha + \beta x_i + \epsilon_i$, where ϵ_i are always assumed to be independently and identically distributed random variables. In this case, if x_i are independently distributed, y_i will be independent to each other as well because y_i only depend on x_i. Nevertheless, in real world, y_i can be affected by its previous observations. For example, there may exist a linear relationship between a company's stock price and its dividend policy. However, current stock price is also affected by previous stock prices. Therefore, we modify the linear regression model by assuming ϵ_i have auto-regressive processes (we only discuss the case of *AR(1)* process), and our model becomes: $y_i = \alpha +$

$\beta x_i + e_i,\ e_i = \rho e_{i-1} + \epsilon_i,\ \epsilon_i \sim iid\ N(0, \delta^2)$.

For estimating α, β, ρ and δ above, we will use ordinary linear square method. Although there exist many other methods, such as maximum likelihood method, the distributions of estimators will be very difficult to derive if those methods are utilized. OLS estimators of α, β, ρ and δ will be derived in this report.

Next, we generate n values $(e_1, e_2, \dots e_n)$ from $AR(1)$ process and n realizations of $X\ (x_1, x_2, \dots x_n)$. We set real values for α, β, ρ and δ. Then we can values of y_i since $y_i = \alpha + \beta x_i + e_i\ (i = 1, 2 \dots n)$. Based on generated sample of $(x_i\ , y_i)$ and our estimators, we may have an estiamte for each $\alpha, \beta, \rho\ and\ \delta$.

The above simulation process will be repeated by 1000 times and we can get 1000 estimates of $\alpha, \beta, \rho\ and\ \delta$. By analyzing and plotting the data, we can explore properties of our estimators.