Tutorial 6

1. For each of the following regression models, indicate whether it is a general linear regression model. If not, state whether it can be expressed in the form of a linear regression model after some suitable transformation

   $a.$ $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 \log X_{i2} + \beta_3 X_{i1}^2 + \varepsilon_i$

   $b.$ $Y_i = \varepsilon_i \exp(\beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2}^2), \quad with\ \varepsilon_i > 0$

   $c.$ $Y_i = \beta_0 \log(\beta_1 X_{i1}) + \varepsilon_i$

   $d.$ $Y_i = \log(\beta_1 X_{i1}) + \beta_2 \log X_{i2} + \varepsilon_i$

   $e.$ $Y_i = [1 + \exp(\beta_0 + \beta_1 X_{i1} + \varepsilon_i]^{-1}$

2. Consider the multiple linear regression models

   $$Y_i = \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i,\ i = 1, ..., n$$

   where ε_i are uncorrelated with $E\varepsilon_i = 0$ and $E\varepsilon_i^2 = \sigma^2$ state the least square criterion and derive the least squares estimators for β_1 and β_2.

3. Consider the multiple regression models

   $$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i1}^2 + \beta_3 X_{i2} + \varepsilon_i,\ i = 1, ..., n$$

   where ε_i are uncorrelated with $E\varepsilon_i = 0$ and $E\varepsilon_i^2 = \sigma^2$. state the least square criterion and derive the least squares normal equations.

4. An analyst wanted to fit the regression model

   $$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i,\ i = 1, ..., n$$

   by the least squares estimation when it is known that $\beta_2 = 4$. How can he implement the calculation in computer code?

5. For the multiple regression model, let r be the sample linear correlation coefficient between Y_i and $\hat{Y}_i$, and R^2 be the multiple determination of the model. Show that $R^2 = r^2$.

6. In a small-scale regression study, the following data were obtained

i:	1	2	3	4	5	6
X_{i1}	7	4	16	3	21	8
X_{i2}	33	41	7	49	5	31
Y_i	42	33	75	28	91	55

   Assume that the multiple linear regression model with independent error terms is appropriate. Using matrix methods, obtain (a) $\mathbf{b}$, (b) $\mathbf{e}$ (c) SR, (d) $\mathbf{s^2(b)}$, (e) $\hat{Y}$ when $X_1 = 10, X_2 = 30$, (f) $s^2(\hat{Y})$ when $X_1 = 10, X_2 = 30$