Tutorial 10

1. Derive the weighted least square normal equations for fitting a **simple** linear regression function when $\sigma_i^2 = kX_i$, where $k > 0$ is a constant.

2. For linear regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + ... + \beta_p X_{ip} + \varepsilon_i, \quad i = 1, ..., n.$$

with

$$Var\left(\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{pmatrix}\right) = \begin{pmatrix} \sigma_1^2 & 0 & ... & 0 \\ 0 & \sigma_2^2 & ... & 0 \\ \cdots & & & \\ 0 & 0 & ... & \sigma_n^2 \end{pmatrix}$$

   (a) If the LSE, b, is used, is the estimator unbiased? what is the variance of the estimated coefficients, $Var(b)$.

   (b) with $w_i = 1/\sigma_i^2$, derive the weighted least square estimator b_w, and calculate $Var(b_w)$

3. For model $Y_i = \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$, if we estimate the coefficients by minimizing

$$Q(b_1, b_2, b_3) = \sum_{i=1}^{n} \{Y_i - b_1 X_{i1} - b_2 X_{i2} - b_3 X_{i3}\}^2 + \lambda(b_1^2 + b_2^2 + b_3^2)$$

   Give the estimator of (b_1, b_2, b_3) in matrix, where $\lambda > 0$ is a constant.

4. For data **data**, with Y and X_1, X_2. Fit a model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i$$

   using WLSE and test whether $H_0 : \beta_1 + \beta_2 = 0$ with $\alpha = 0.05$.

5. for data **data**

X	Y	X	Y
1	1.2301	2	-0.1862
1	1.3826	3	7.2876
1	0.7203	3	2.3073
1	1.2084	3	13.4502
2	1.8623	3	-0.2807
2	2.0064	3	2.6161
2	4.5815		

(a) If we use weighted least squares estimation, how to assign the weight?

(b) Compare the estimated models based on LSE and WLSE, what is your observations.