Stock market prediction and unequal variance

Stock market index is a useful indicator for economy and investment. Its prediction is obviously very useful. BUT

1. Mathematically, the movement of the index is a "random walk"

$$y_t = y_{t-1} + \varepsilon_t$$

   where ε_t are IID random variables.

2. Please compare a random walk with the Dow Jones index (from 1999-2009) **(data)**, and observe their similarity

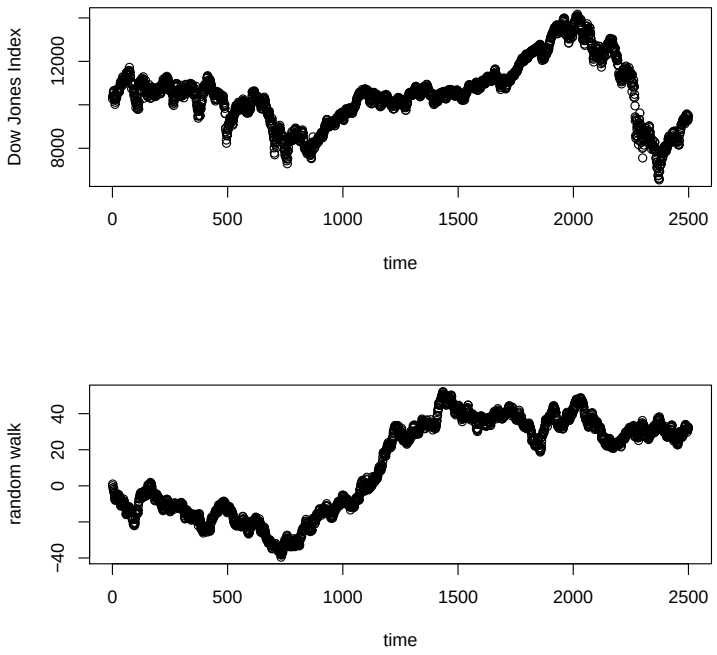

Figure 1: **(R code)**

3. The return is defined as

$$r_t = y_t - y_{t-1},$$

   or in practice, more conveniently

$$R_t = \log(y_t) - \log(y_{t-1}).$$

4. the return is not predictable by its past values, since

$$r_t = y_t - y_{t-1} = \varepsilon_t$$

or (if we want to predict r_t based on y_{t-1})

$$r_t = \underbrace{0 + 0 * y_{t-1}}_{\text{nothing predicted}} + \varepsilon_t$$

BUT ....

1. the practical data has UNEQUAL variance,

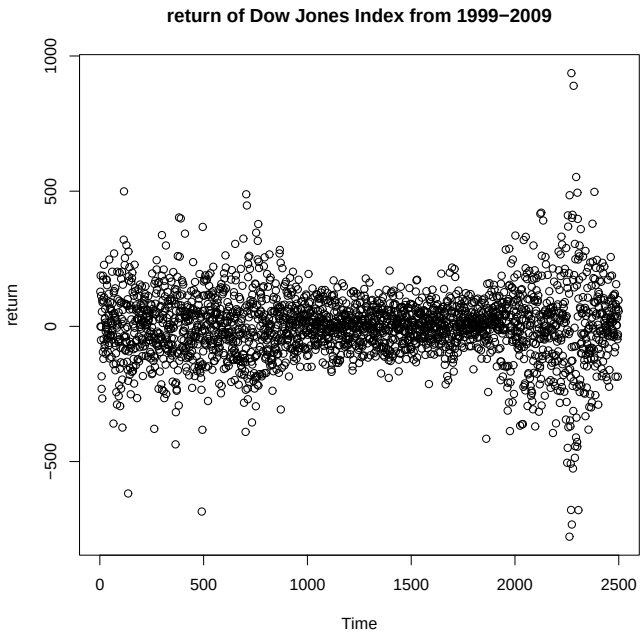

Figure 2: **(R code)**

leading to a model

$$r_t = \sigma_t \epsilon_t$$

where σ_t is a function of time t. How to model σ_t is a big issue in econometrics. The Nobel Prize of Economics in 2003 is the ARCH model (Robert F. Engle)

$$\sigma_t^2 = \beta_0 + \beta_1 r_{t-1}^2 + \xi_t$$

2. Risk management: recall what is the statistical meaning of variance