Tutorial 6

1. Suppose $m(x_1, ..., x_p) = \phi(\alpha_1 x_1 + ... + \alpha_p x_p)$. Prove that

$$\begin{pmatrix} \frac{\partial m(x_1,...,x_p)}{\partial x_1} \\ \frac{\partial m(x_1,...,x_p)}{\partial x_2} \\ \cdots \\ \frac{\partial m(x_1,...,x_p)}{\partial x_p} \end{pmatrix} = \phi'(\alpha_1 x_1 + ... + \alpha_p x_p) \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_p \end{pmatrix}$$

2. For a single-index model $Y = \phi(\alpha^\top X) + \varepsilon$. Suppose (X_i, y_i) are the observations and the estimator for α is $\hat{\alpha}$. For a new point X', predict the expectation of response using local linear kernel smoothing.

3. For the ozone concentration **data**.

   (a) fit a linear regression model and predict the ozone level when radiation=184.8, temperature=77.8, wind =9.9.

   (b) fit a partially linear regression model

   $$ozone = \beta_1 * termperature + \beta_2 * Wind + g(Radiation) + \varepsilon.$$

   predict the ozone level when radiation=184.8, temperature=77.8, wind =9.9.

   (c) fit a single-index model and predict the ozone level when radiation=184.8, temperature=77.8, wind =9.9. and what is the prediction confidence interval for the expectation of Y.

4. For **data A** (the first two columns are independent variables, the last one response variable), fit a linear regression model; check whether the model is adequate. If not, propose a new parametric model.

5. For **data B**, fit a single-index model

$$Y = g(\beta_1 X_1 + ... + \beta_6 X_6) + \varepsilon$$

and check which covariates can be removed