ST4241: Design and Analysis of Clinical Trials

2009/2010: Semester I

Tutorial 1

1. The following three questions pertain to different aspects of the multiple comparison artifact.

(a) If each of K contrasts was tested for significance at the α significance level, how many contrasts would you expect, just by chance, to find significant? What is this expected number when $K = 100$ and $\alpha = 0.05$?

(b) Suppose that the ratios $\hat{C}/\text{se}(\hat{C})$ for each of K contrasts are mutually independent and that each is tested for significance at the α significance level. Show that the probability that at least one is erroneously found to be significant is equal to $1 - (1 - \alpha)^K$. What is this probablity if $K = 14$ and $\alpha = 0.05$?

(c) Investigators will occasionally single out the samallest and the largest of g means for a t test at the α significance level. That is, when the sample sizes are all equal to a common n, the largest and smallest means are declared to differ significantly if

$$L = \frac{\max \bar{X}_i - \min \bar{X}_i}{\sqrt{\text{WMS}}} \sqrt{\frac{n}{2}}$$

exceeds $t_{n.-g,\alpha/2}$. The correct critical values of L are given by the entries in Table A.5 of Fleiss divided by $\sqrt{2}$. Suppose that $g = 4, n = 7$ and $L = 2.1$. What would the verdict be, significant or not, if L was compared to $t_{24,0.0025}$? Check that L would have to exceed 2.76 in order for significance to be declared using the correct test criterion.

2. Prove that when $g = 2$ the ratio $F = \text{BMS}/\text{WMS}$ of mean squares from the analysis of variance table is identical to the square of the t ratio given by

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s} \sqrt{\frac{n_1 n_2}{n_1 + n_2}},$$

where

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}.$$

3. The following table gives the 12 means and standard deviations of the survival times of 12 groups of animals, each receiving a combination of poisons and treatment.

Group	$\bar{X}_i$	s_i
1	4.125	0.695
2	8.800	1.608
3	5.675	1.567
4	6.100	1.128
5	3.200	0.753
6	8.150	3.363
7	3.750	0.569
8	6.675	2.710
9	2.100	0.216
10	3.350	0.465
11	2.350	0.129
12	3.250	0.265

(i) Confirm that, of the values $k = 1/2, 1, 2$ and 3, the smallest ratio

$$R_k = \frac{\max(\frac{s_i}{\bar{X}_i^k})}{\min(\frac{s_i}{\bar{X}_i^k})}$$

is achieved when $k = 2$, thus indicating that s is more nearly proportial to $\bar{X}^2$ than to the other powers of $\bar{X}$.

(ii) Can you suggest any other way to determine an appropriate transformation of the original data (which is not given above)?

4. Lysozyme levels in the gastric juice of 29 patients with peptic ulcer and of 30 normal controls are given below.

Patient group:

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 0.2  0.3  0.4  1.1  2.0  2.1  3.3  3.8  4.5  4.8  4.9   5.0  5.3  7.5  9.8 10.4
10.9 11.3 12.4 16.2 17.6  18.9  20.7 24.0 25.4 40.0 42.2 50.0 60.0
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Control group:

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 0.2  0.3  0.4  0.7  1.2  1.5 1.5 1.9  2.0  2.4  2.5  2.83.6  4.8  4.8  5.4  5.7
5.8  7.5  8.7  8.8  9.1 10.3 15.6  16.1 16.5 16.7 20.0 20.7 33.0
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(i) Apply t test to the transformed values of $Y = \ln(X)$ to compare the two groups above. Compare the result with that of the t test applied to the original data. Make comments on the comparison.

(ii) Apply Mann-Whitney-Wilcoxon test to the above data. Are the two groups significantly different at level α according to this test?